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ASSESSMENT and QUALIFICATIONS ALLIANCE

## General Certificate of Education

# Mathematics 6360

## MFP3 Further Pure 3

# Mark Scheme

*2007 examination - January series*

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.



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## Key to mark scheme and abbreviations used in marking

| | |
|---|---|
| M | mark is for method |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |

| | | | |
|---|---|---|---|
| √or ft or F | follow through from previous incorrect result | MC | mis-copy |
| CAO | correct answer only | MR | mis-read |
| CSO | correct solution only | RA | required accuracy |
| AWFW | anything which falls within | FW | further work |
| AWRT | anything which rounds to | ISW | ignore subsequent work |
| ACF | any correct form | FIW | from incorrect work |
| AG | answer given | BOD | given benefit of doubt |
| SC | special case | WR | work replaced by candidate |
| OE | or equivalent | FB | formulae book |
| A2,1 | 2 or 1 (or 0) accuracy marks | NOS | not on scheme |
| $-x$ EE | deduct $x$ marks for each error | G | graph |
| NMS | no method shown | c | candidate |
| PI | possibly implied | sf | significant figure(s) |
| SCA | substantially correct approach | dp | decimal place(s) |

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

**MFP3**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 1(a) | $y\ (1.05) = 0.6 + 0.05 \times [\ln(1+1+0.6)]$ | M1A1 | | |
| | $= 0.6477(7557..) = 0.6478$ to 4dp | A1 | 3 | Condone >4 dp |
| (b) | $k_1 = 0.05 \times \ln(1+1+0.6) = 0.0477(75\ldots)$ | M1 | | PI |
| | | A1F | | ft candidate's evaluation in (a) |
| | $k_2 = 0.05 \times \text{f}\,(1.05,\ 0.6477\ldots)$<br>$\ldots = 0.05 \times \ln\left(1+1.05^2+0.6477\ldots\right)\big]$ | M1 | | |
| | $\ldots = 0.0505(85\ldots)$ | A1F | | PI |
| | $y(1.05) = y(1) + \frac{1}{2}[k_1 + k_2]$<br>$= 0.6 + 0.5 \times 0.09836\ldots$ | m1 | | Dep on previous two Ms and numerical values for $k$'s |
| | $= 0.6492$ to 4dp | A1F | 6 | Must be 4 dp… ft one slip |
| | **Total** | | **9** | |
| 2 | $r - r\sin\theta = 4$ | M1 | | |
| | $r - y = 4$ | B1 | | $r\sin\theta = y$ stated or used |
| | $r = y + 4$ | A1 | | |
| | $x^2 + y^2 = (y+4)^2$ | M1 | | $r^2 = x^2 + y^2$ used |
| | $x^2 + y^2 = y^2 + 8y + 16$ | A1F | | ft one slip |
| | $y = \dfrac{x^2 - 16}{8}$ | A1 | 6 | |
| | **Total** | | **6** | |
| 3(a) | IF is $\exp\left(\int \frac{2}{x}\,\text{d}x\right)$ | M1 | | And with integration attempted |
| | $= \text{e}^{2\ln x}$ | A1 | | |
| | $= x^2$ | A1 | 3 | CSO **AG** be convinced |
| (b) | $\frac{\text{d}}{\text{d}x}\left[yx^2\right] = 3x^2(x^3+1)^{\frac{1}{2}}$ | M1A1 | | PI |
| | $\Rightarrow yx^2 = \frac{2}{3}\left(x^3+1\right)^{\frac{3}{2}} + A$ | m1 | | $k\left(x^3+1\right)^{\frac{3}{2}}$ |
| | | A1 | | Condone missing '$A$' |
| | $\Rightarrow 4 = \frac{2}{3}(9)^{\frac{3}{2}} + A$ | m1 | | Use of boundary conditions to find constant |
| | $\Rightarrow A = -14$ | | | |
| | $\Rightarrow y = x^{-2}\left\{\frac{2}{3}\left(x^3+1\right)^{\frac{3}{2}} - 14\right\}$ | A1 | 6 | Any correct form |
| | **Total** | | **9** | |

**MFP3 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **4(a)** | Integrand is not defined at $x = 0$ | E1 | 1 | OE |
| **(b)** | $\int x^{-\frac{1}{2}} \ln x \, dx = 2x^{\frac{1}{2}} \ln x - \int 2x^{\frac{1}{2}} \left(\frac{1}{x}\right) dx$ | M1 | | $... = kx^{\frac{1}{2}} \ln x \pm \int f(x)$, with $f(x)$ not involving the 'original' $\ln x$ |
| | | A1 | | |
| | $...... = 2x^{\frac{1}{2}} \ln x - 4x^{\frac{1}{2}} (+c)$ | A1 | 3 | Condone absence of '$+ c$' |
| **(c)** | $\int_0^{e} \frac{\ln x}{\sqrt{x}} dx = \lim_{a \to 0} \int_a^{e} \frac{\ln x}{\sqrt{x}} dx$ | M1 | | |
| | $= -2e^{\frac{1}{2}} - \lim_{a \to 0} \left[ 2a^{\frac{1}{2}} \ln a - 4a^{\frac{1}{2}} \right]$ | M1 | | $F(b) - F(a)$ |
| | But $\lim_{a \to 0} a^{\frac{1}{2}} \ln a = 0$ | B1 | | Accept a general form e.g. $\lim_{x \to 0} x^k \ln x = 0$ |
| | So $\int_0^{e} \frac{\ln x}{\sqrt{x}} dx$ exists and $= -2e^{\frac{1}{2}}$ | A1 | 4 | |
| | **Total** | | **8** | |
| **5** | Auxl. eqn $m^2 - 4m + 3 = 0$ | M1 | | PI |
| | $m = 3$ and 1 | A1 | | PI |
| | CF is $A e^{3x} + B e^{x}$ | A1F | | |
| | PI Try $y = a + b \sin x + c \cos x$ | M1 | | Condone '$a$' missing here |
| | $y'(x) = b \cos x - c \sin x$ | A1 | | |
| | $y''(x) = -b \sin x - c \cos x$ | A1F | | ft can be consistent sign error(s) |
| | Substitute into DE gives | M1 | | |
| | $a = 2$ | B1 | | |
| | $4c + 2b = 5$ and $2c - 4b = 0$ | A1 | | |
| | $b = 0.5$, | A1F | | ft a slip |
| | $c = 1$ | A1F | | ft a slip |
| | GS: $y = A e^{3x} + B e^{x} + 2 + 0.5\sin x + \cos x$ | B1F | 12 | $y$ = candidate's CF and candidate's PI (must have exactly two arbitrary constants) |
| | **Total** | | **12** | |

**MFP3 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **6(a)(i)** | $f'(x)=\frac{1}{2}(1+2x)^{-\frac{1}{2}}(2)=(1+2x)^{-\frac{1}{2}}$ | M1A1 | | |
| | $f''(x)=-(1+2x)^{-\frac{3}{2}}$ | A1F | | ft a slip |
| | $f'''(x)=3(1+2x)^{-\frac{5}{2}}$ | A1 | 4 | |
| **(ii)** | $f(x)=(1+2x)^{\frac{1}{2}} \Rightarrow f(0)=1;$ | B1 | | |
| | $f'(0) = 1;\ f''(0) = -1;\ f'''(0) = 3$ | M1 | | All three attempted |
| | | A1F | | ft on $k(1+2x)'''$ |
| | $f(x) = f(0) + xf'(0) + \frac{x^2}{2}f''(0) + \frac{x^3}{6}f'''(0)$ | | | |
| | $\ldots \approx 1+x-\frac{x^2}{2}+\frac{x^3}{2}$ | A1 | 4 | CSO **AG** |
| **(b)** | $e^x(1+2x)^{\frac{1}{2}} \approx$ | | | |
| | $\left(1+x+\frac{x^2}{2}+\frac{x^3}{6}\right)\left(1+x-\frac{x^2}{2}+\frac{x^3}{2}\right)$ | M1 | | Attempt to expand needed |
| | $\approx 1+x(1+1)+x^2(-0.5+1+0.5)$ | A1 | | |
| | $+\ x^3\left(\frac{1}{2}-\frac{1}{2}+\frac{1}{2}+\frac{1}{6}\right)$ | | | |
| | $\approx 1+2x+x^2+\frac{2}{3}x^3$ | A1 | 3 | CSO |
| **(c)** | $e^{2x} = 1+2x+\frac{(2x)^2}{2}+\frac{(2x)^3}{6}+\ldots$ | B1 | 1 | |
| | $= 1+2x+2x^2+\frac{4}{3}x^3+\ldots$ | | | |
| **(d)** | $1-\cos x=\frac{1}{2}x^2+\{o(x^4)\}$ | B1 | | |
| | $\frac{e^x(1+2x)^{\frac{1}{2}}-e^{2x}}{1-\cos x} =$ | | | |
| | $\frac{1+2x+x^2+\frac{2}{3}x^3-\left[1+2x+2x^2+\frac{4}{3}x^3\right]}{\frac{1}{2}x^2+\{o(x^4)\}}$ | M1 | | Series used |
| | $\lim_{x\to 0}\ldots = \lim_{x\to 0}\frac{-x^2+\{o(x^3)\}}{\frac{1}{2}x^2+\{o(x^4)\}} =$ | A1F | | |
| | $\lim_{x\to 0}\frac{-1+o(x)}{\frac{1}{2}+o(x^2)} = -2$ | A1F | 4 | ft a slip but must see the intermediate stage |
| | **Total** | | **16** | |

**MFP3 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **7(a)** | Area = $\frac{1}{2}\int(6+4\cos\theta)^2\,\mathrm{d}\theta$ | M1 | | use of $\frac{1}{2}\int r^2\,\mathrm{d}\theta$ |
| | $=\frac{1}{2}\left(\int_{-\pi}^{\pi}36+48\cos\theta+16\cos^2\theta\right)\mathrm{d}\theta$ | B1<br>B1 | | for correct expansion of $[6+4\cos\theta]^2$<br>for limits |
| | $=\left(\int_{-\pi}^{\pi}18+24\cos\theta+4(\cos 2\theta+1)\right)\mathrm{d}\theta$ | M1 | | Attempt to write $\cos^2\theta$ in terms of $\cos 2\theta$ |
| | $=\left[22\theta+24\sin\theta+2\sin 2\theta\right]_{-\pi}^{\pi}$ | A1F | | correct integration ft wrong coefficients |
| | $= 44\pi$ | A1 | 6 | CSO |
| **(b)** | At $P$, $r=4$; At $Q$, $r=2$; | B1 | | PI |
| | $P\ \{x=\}\ r\cos\theta=4\cos\frac{2\pi}{3}=-2$ | M1 | | Attempt to use $r\cos\theta$ |
| | $Q\ \{x=\}\ r\cos\theta=2\cos\pi=-2$ | A1 | | Both |
| | Since $P$ and $Q$ have same *'x'*, $PQ$ is vertical so $QP$ is parallel to the vertical line $\theta=\frac{\pi}{2}$ | E1 | 4 | |
| **(c)(i)** | $OP = 4$; $OS = 8$; | B1 | | |
| | Angle $POS=\frac{\pi}{3}$ | B1 | | or $S\,(4, 4\sqrt{3})$ and $P\,(-2, 2\sqrt{3})$ |
| | $PS^2=4^2+8^2-2\times4\times8\times\cos\frac{\pi}{3}$ oe | M1 | | Cosine rule used in triangle $POS$<br>OE $PS^2=(4+2)^2+(4\sqrt{3}-2\sqrt{3})^2$ |
| | $PS=\sqrt{48}\ \ \{=4\sqrt{3}\}$ | A1 | 4 | |
| **(ii)** | Since $8^2=4^2+\left(\sqrt{48}\right)^2$,<br>$OS^2=OP^2+PS^2\Rightarrow OPS$ is a right angle. (Converse of Pythagoras Theorem) | E1 | 1 | Accept valid equivalents e.g.<br>$PR = 2PQ = 2(2\sqrt{3}) = PS$.<br>$\angle SRP=\angle RSP=\angle RPO=\frac{\pi}{6}$<br>$\Rightarrow$ $OPS$ is a right angle |
| | **Total** | | **15** | |
| | **TOTAL** | | **75** | |